Universal Joint Source–Channel Coding Under an Input Energy Constraint

Omri Lev

Joint work with Anatoly Khina

Dept. EE–Systems, Tel Aviv University, Tel Aviv, Israel

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JSCC over AWGN Channel with Unlimited Bandwidth (BW)

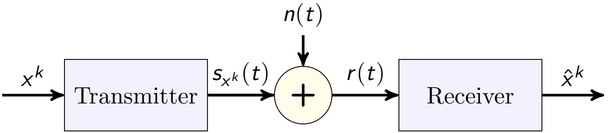
Source: $x^k \in \mathbb{R}^k$—i.i.d. entries with density $f_x(a)$

Transmitter: $s_{x^k}(t)$

- Input energy constraint E: $\int_{-\frac{kT}{2}}^{\frac{kT}{2}} \left|s_{x^k}(t)\right|^2 dt \leqslant kE, \quad \forall x^k$

Channel: $r(t) = s_{x^k}(t) + n(t)$

- n—continuous-time AWGN with power spectral density $N/2$



- Power limited regime ($BW \to \infty$): $C = \frac{PT}{N} = \frac{E}{N} \triangleq \mathrm{ENR}$

What is the minimum distortion $D \triangleq \frac{1}{k}\mathbb{E}\left[\left\|x^k - \hat{x}^k\right\|^2\right]$?

Problem Formulation: Unknown ENR

- Only the receiver knows the noise level N
  - The transmitter accommodate for multiple noise levels
- **Main goal**: Design $s_{x^k}(t)$ that achieves

$$D \leqslant \sigma_x^2 \mathscr{F}(N), \quad \forall N \geqslant 0$$

with minimum energy.

  - We concentrate on polynomial profiles:

$$\mathscr{F}(N) \triangleq \frac{1}{1 + \left(\frac{\tilde{E}}{N}\right)^L}, \tilde{E} \in \mathbb{R}^+$$

- Tightest lower bound for $L = 2$ [Baniasadi–Tuncel '21]:

$$E \geqslant 0.9057 \tilde{E}$$

Part I

Background

Hybrid Digital–Analog Schemes [Köken–Tuncel '17]

- Multiple transmission layers — layer k optimized for N_k
- Receiver combines information from different layers
- k'th layer parameters and energy optimized such that

$$D(N) \leqslant \mathscr{F}(N), N_k < N \leqslant N_{k+1}$$

Layered Transmission for AWGN With $W \to \infty$

- Transmitter:
  - Uncoded transmission: $Y_0 = \sqrt{E_0} X + N_0$
  - Digital transmission: $S_k = Q\left(e_k\right), e_k = S_{k-1} - Q\left(S_{k-1}\right)$
- Receiver:
  1. Digital: Decodes $\{S_k\}_{k=1}^{\ell}$, ℓ is determined from noise level
  2. Analog: Estimates $\hat{e}_\ell$ from $\left\{\hat{S}_k\right\}_{k=1}^{\ell}$ and Y_0, to generate $\hat{X}$
- Total energy: $E_{\min} \cong 3.184\tilde{E}$ (LB: $E_{\min} \geqslant 0.905\tilde{E}$)

Hybrid Digital–Analog Schemes [Baniasadi–Tuncel '20]

Improvement idea

- Only the first layer improves with the ENR
- Use multiple uncoded layers to boost analog part performance

Scheme

- Transmitter:
  - Uncoded transmissions: $Y_k = \sqrt{E_k} e_k + N_k$, $e_0 = X$
  - Digital transmission: $S_k = Q(e_k), e_k = S_{k-1} - Q(S_{k-1})$
- Receiver:
  1. Digital: Decodes $\{S_k\}_{k=1}^{\ell}$, ℓ is determined from noise level
  2. Analog: Estimates $\hat{e}_\ell$ from $\left\{\hat{S}_k, Y_k\right\}_{k=1}^{\ell}$, to generate $\hat{X}$
- Total energy: $E_{\min} \cong 2.32\tilde{E} < 3.18\tilde{E}$ (LB: $E_{\min} \geqslant 0.905\tilde{E}$)

Hybrid Digital–Analog Schemes Limitations

Drawbacks

- Linear transmission—inefficient use of bandwidth (BW)
- Performance saturation:
  - Quantization does not improve with ENR
  - Uncoded transmissions—improve only *linearly*

Finite BW: Modulo-lattice modulation (MLM) [Reznic–Feder–Zamir ’06]

Unknown ENR: Modulo-based Wyner–Ziv coding outperforms successive refinement based schemes.

Our Approach: Two improvements

1. Use Modulo-based Wyner–Ziv in infinite BW setting
2. Replace linear transmissions with Non-linear JSCC

Part II

M-Layer Scheme

First Ingredient: Modulo Lattice Modulation (MLM)

- Source comprises known (j^k) and unknown (q^k) parts

$$x^k = q^k + j^k$$

Transmitted over an additive channel:

$$y^k = m^k\left(x^k\right) + z^k, \quad \frac{\frac{1}{k}\mathbb{E}\left[\left\|m^k\right\|^2\right]}{\sigma_z^2} \triangleq \mathrm{SNR}$$

- **Transmitter:**

$$m^k = [\eta x^k + d^k]_\Lambda$$

  - $d^k \sim \mathrm{Unif}\left(\mathscr{V}_0\right)$, dither signal

- **Receiver:**

$$\tilde{y}^k = [\alpha_c y^k - \eta j^k - d^k]_\Lambda = [\eta q^k + z^k_{\mathrm{eff}}]_\Lambda,$$

$$\hat{x}^k = \frac{\alpha_s}{\eta}\tilde{y}^k + j^k$$

First Ingredient: Modulo-Lattice Modulation (MLM)

MLM Performance [Reznic et al. '06, Kochman–Zamir '09, Ordentlich–Erez '16]

Let q^k and z^k be *semi norm-ergodic* sequences. Then,

$$D \leqslant \frac{\sigma_q^2}{1+\mathrm{SNR}} \triangleq D^*$$

- a^k semi norm-ergodic if:

$$\Pr\left(a^k \notin \mathscr{B}\left(0, \sqrt{(1+\delta)\, k\sigma_a}\right)\right) \leqslant \epsilon$$

Universality [Kochman–Zamir '09, Ordentlich–Erez '16]

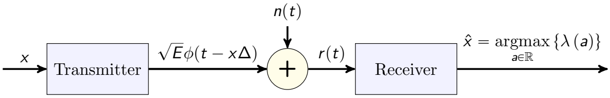
- $\mathrm{SNR} \geqslant \mathrm{SNR}_0$, $\sigma_q^2 \leqslant \tilde{\sigma}_q^2$
- Tx oblivious of SNR and σ_q^2, knows SNR_0 and $\tilde{\sigma}_q^2$

$$D \leqslant \min\left\{\frac{\sigma_q^2}{1+\mathrm{SNR}_0}, \frac{\tilde{\sigma}_q^2}{1+\mathrm{SNR}} \cdot \frac{1+\mathrm{SNR}_0}{\mathrm{SNR}_0}\right\}$$

- For $\mathrm{SNR} \geqslant \mathrm{SNR}_0 \gg 1$: $D \to \frac{\sigma_q^2}{1+\mathrm{SNR}} = D^*$

Second Ingredient: Analog PPM [Lev–Khina '21]

- Transmit a rectangular pulse with width $1/\beta$
- Pulse delay = current source realization
- Estimate delay using Maximum A-Posteriori (MAP) estimator



- $\lambda(a) = \frac{2\sqrt{E}}{N}\int_{-\infty}^{\infty} r(t)\phi(t - a\Delta)dt + \log\left(f_x(a)\right)$

Advantages

- No quantization — fully analog transmission:
  1. Distinction between close source values (transmitter-side "soft-decision")
  2. Small noise values — small distortion

Second Ingredient: Analog PPM [Lev–Khina '21]

Performance

Performance guarantees for uniform and Gaussian sources:

$$D \leqslant K_1 \cdot \mathrm{ENR}^{-K_2} \cdot e^{-\mathrm{ENR}/3}$$

- Achieves Burnashev's exponential decay
- Achieves additional polynomial decay
- Outperform other existing schemes

Robustness

- Pulse width β optimized for specific ENR_0 point
- For $\mathrm{ENR} > \mathrm{ENR}_0$ the performance decay *quadratically*
  This is beneficial for universal schemes!

M-Layer Scheme

"Black-box" approach:

- **Generate set of layers using modulo-encoding (MLM)**
- **Transmit independently using scalar JSCC scheme**

Equivalent Channel

- MLM operates over the channel $m_i^k \rightarrow \hat{m}_i^k$
  1. Equivalent noise is JSCC output noise
  2. Noise not necessarily Gaussian

M-Layer Scheme: Remarks

Interleaving

- Interleaving is used between MLM and JSCC steps
  - Receiver applies de-interleaving between JSCC and MLM
- Ensures independence between vector entries
- Allows to use MLM results for semi norm-ergodic variables

Gaussianization [AMIMON '07, No–Weissman '16, Erez–Hadad '16]

- PPM inputs multiplied by orthogonal matrix
- Allows to use analysis of JSCC with Gaussian inputs
- Receiver multiplies by $H^{-1} = H^T$—Gaussianizes the noise

- Dimension k is general—closed form solutions for $k \to \infty$

M-Layer Scheme: Linear Layers

Performance

Let $L > 1$, $\tilde{E} > 0$ and $k \to \infty$. Then, polynomial profile is achievable for any transmit energy E that satisfies

$$E > \delta_{\mathrm{lin}}(L)\,\tilde{E},$$

where

$$\delta_{\mathrm{lin}}(L) \triangleq \frac{1}{2} \cdot \min_{(\alpha,x)\in\mathbb{R}_+^2} \left\{ \left(\frac{\mathrm{e}^{\alpha}}{x}\right)^{L-1} + \frac{x}{2}\left(\mathrm{e}^{\alpha L} - 1\right)\left(1 + \sqrt{1 + \frac{4\mathrm{e}^{\alpha(L+1)}}{\left(1-\mathrm{e}^{\alpha L}\right)^2}}\right)\frac{\mathrm{e}^{-2\alpha}}{1-\mathrm{e}^{-\alpha}} \right\}$$

For $L = 2$, we get $E_{\min} \cong 2.16\tilde{E} < 2.32\tilde{E}$ (LB: $E_{\min} \geqslant 0.905\tilde{E}$)

- Outperform previous hybrid digital–analog schemes
- Achieved using MLM (no PPM yet)

M-Layer Scheme: PPM

Performance

Let $L = 2$, $\tilde{E} > 0$ and $k \to \infty$. Then, polynomial profile is achievable for any transmit energy E that satisfies

$$E_{\min} \cong 1.961\tilde{E} < 2.16\tilde{E} \qquad (\text{LB: } E_{\min} \geqslant 0.905\tilde{E})$$

- Offers further improvement over linear-based transmitter
- PPM bounds are not tight—**improvement is even higher**

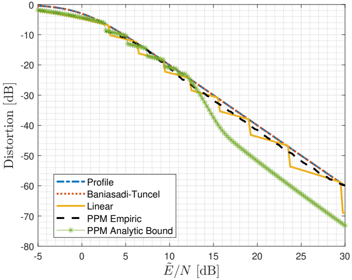


Figure: Distortion

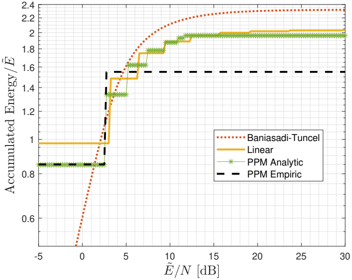


Figure: Accumulated Energy

M-Layer Scheme: Scalar

- Numerical simulation of uniform source with $k = 1$
- Quadratic profile

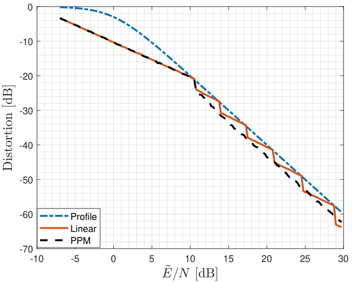


Figure: Distortion

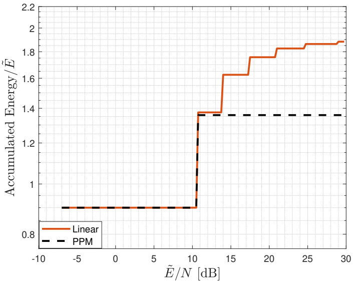


Figure: Accumulated Energy

- Scheme applies for practical low-delay scenario
  - **PPM allows saving of transmitted energy**

Summary

Summary

- We introduced robust energy-efficient communication scheme
  1. Based on MLM and JSCC
  2. Analog PPM offers performance boost
- Total energy is lower than best reported results
  - Real improvement is even higher than theoretical bounds
- Practical low-delay scenario simulation results presented

Future Research

- Improving the optimization can close gap to outer bound
- Different direction: improve further outer bound [Köken–Tuncel '17] [Baniasadi–Tuncel '20]
- Universal SI at the receiver [Baniasadi–Tuncel '21]
- Dual problem: near-zero bandwidth [Baniasadi–Tuncel '20]